HP 12c Financial Calculator - Using the RPN Stack to Solve Problems Efficiently

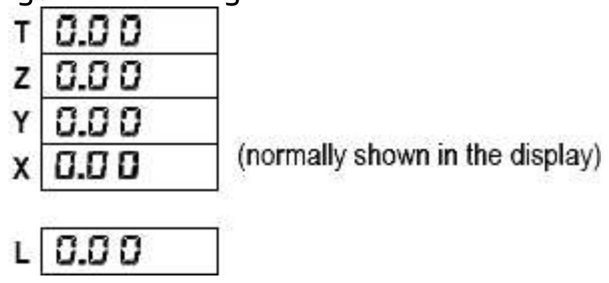
The benefits of understanding RPN stack operations

RPN stands for Reverse Polish Notation. Anytime the simplest operation is performed in the HP 12c, many of these RPN resources are used. Mastering RPN leads to an enhanced performance when using the calculator. The first step to master RPN usage is to know all of its available resources.

Understanding the HP 12c RPN stack operation

When in normal, 'run' mode, every operation performed in the HP 12c uses the display contents or places results on it. The display always shows the contents of the X-register. A register is a predefined place in the calculator memory that is able to hold a formatted number with a ten-digit mantissa and a two-digit exponent of ten. The X-register is one of five registers that form the RPN stack, represented in Figure 1 with all contents cleared to zero.

Figure : Stack registers



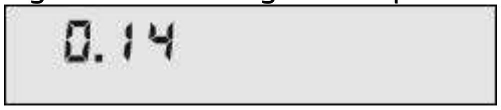
Anytime a number is keyed in, the X-register is updated to hold this number. It is then available for use.

Example 1

What is the most common keystroke sequence to compute the reciprocal of 7.2? What registers in the stack are used?

Solution

The most common keystroke sequence to compute the reciprocal of 7.2 is:

Keystroke	Display
7 • 2 1/x	Figure : Calculating the reciprocal of 7.2 

In this case, simply typing in the number and pressing the related key are enough to compute the resulting value. There are other functions that use only the X-register contents, like g LN or g √x. These are the 'one-number functions', and when they are pressed, the calculator proceeds with the following operation sequence:

1. Take a copy of the X-register contents and put it in L-register (last-x);
2. Replace actual X-register contents with the resulting value.

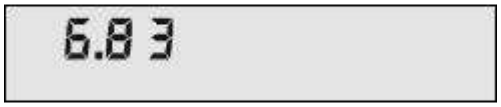
If a second number needs to be typed in right after the first one to be used with it, they must be separated with the ENTER key. For example, to add 3 and 5 together, you would press 3 ENTER 5 +.

Example 2

What is the most common keystroke sequence to enter 2.33 and 4.5 and add them to each other? What registers in the stack are used?

Solution

The most common keystroke sequence to add 2.33 to 4.5 is:

Keystroke	Display
4 • 5 ENTER 2 • 3 3 +	Figure : Entering and adding the values of 2.33 and 4.5  6.83

In fact, all of the stack registers have their contents changed during this single addition. However, the real question is that since both numbers must "exist" in the calculator memory before + is pressed, "Where do they exist?" When + is pressed, the calculator proceeds with the following operation sequence:

1. Take a copy of the X-register contents and put it in L-register (last-x);
2. Take Y-register contents and add to X-register contents;
3. Replace actual X-register contents for resulting value;
4. "Drop" T- and Z-register contents respectively to Z- and Y-registers;
5. Maintain a copy of T-register contents in T.

Based on these facts, it is seen that ENTER not only separates two numbers entered in a keystroke sequence, but it actually provides that a copy of the number currently in the X-register is preserved for immediate use, and this copy is kept in the Y-register. This way, when + is pressed in this example, the number that is in the Y-register is the one that was copied there by ENTER. This same operational sequence is observed when −, X, ÷ and y^x are used. These are the 'two-number functions'. When the ENTER key is pressed, the copy of the number shown in the display is actually kept in the Y-register. This pushes the previous contents of the Z- and Y-registers up to the T- and Z-registers, respectively. This is called "stack lifting" because the stack register contents are "lifted" or pushed up. Previous T-register contents are lost after ENTER is pressed, since they are "pushed" off the top of the stack.

Viewing and reordering stack registers contents

Example 3

How would you fill the T, Z, Y and X stack registers with the numbers 44, 33, 22 and 11, respectively?

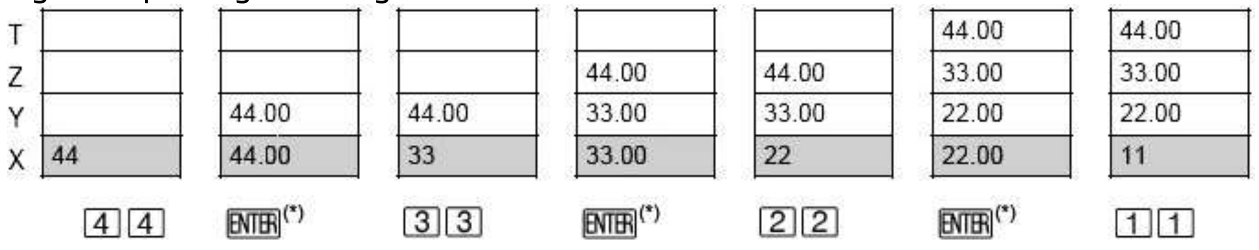
Solution

The most common keystroke sequence to fill the stack registers with these numbers is:

4 4 ENTER 3 3 ENTER 2 2 ENTER 1 1

The stack registers contents would be updated according to the diagrams shown below:

Figure : Updating stack register's contents

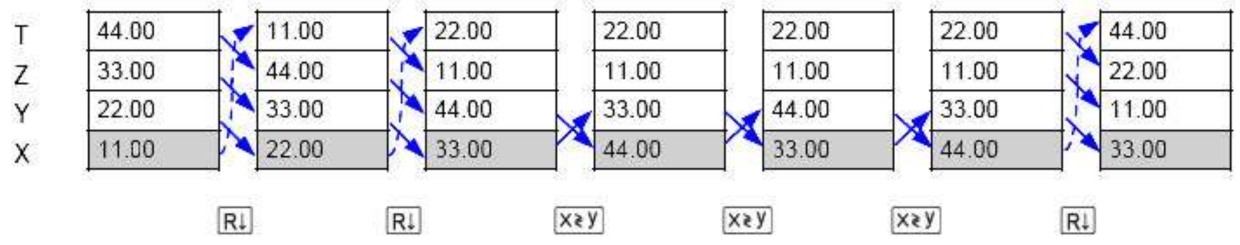


NOTE:

Entering a number in the X-register right after ENTER, CLx, Σ+ or g Σ- overwrites current X-register contents and does not lift stack contents.

Under certain circumstances, keeping track of whatever is in each stack register is not practical for daily, quick computations. To help viewing the stack contents, two functions are an aid: x≷y (X exchanges Y contents) and R↓ (roll-down all stack registers contents). Keeping previous contents as they are, let us try the following keystroke:

Figure : Viewing the stack register contents by making use of the functions



Using Last-x contents in chain calculations

The L-register is automatically updated whenever the X-register contents are changed. In this case, the L-register is loaded with a copy of the last value in X prior to executing the function, hence the reference Last x. Whenever possible, using L-register contents in chain calculations avoids numbers or intermediate results to be typed in again. The L-register's contents can be retrieved to the X-register anytime it is needed with the sequence $\boxed{g}\boxed{LSTx}$.

Example 4

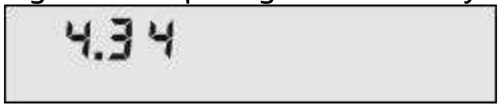
What is the shortest keystroke sequence to find 'y' value in the following expression given x=3.4567?

Figure : Equation to find the value of y

$$y = \sqrt{x} + 2 \times \ln(x)$$

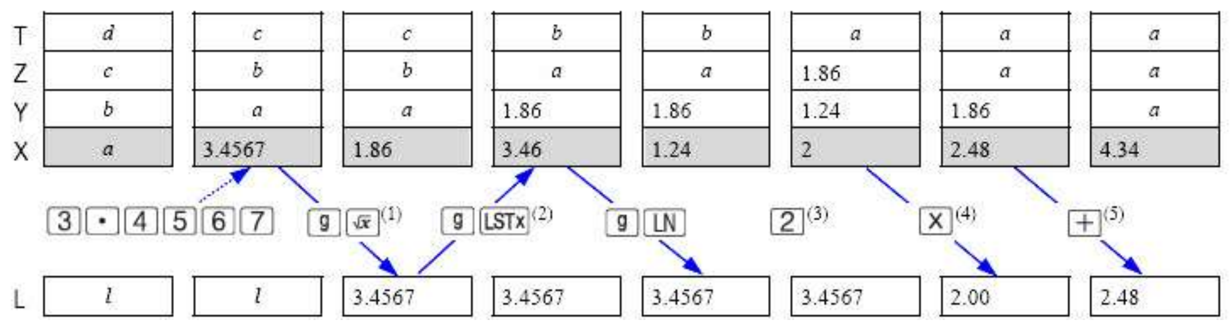
Solution

The shortest keystroke sequence to compute 'y' is:

Keystroke	Display
$\boxed{3}\boxed{\bullet}\boxed{4}\boxed{5}\boxed{6}\boxed{7}\boxed{g}\boxed{\sqrt{x}}$ $\boxed{g}\boxed{LSTx}\boxed{g}\boxed{LN}\boxed{2}\boxed{\times}\boxed{+}$	Figure : Computing the value of y 

When $\boxed{g}\boxed{\sqrt{x}}$ is pressed after 3.4567 is keyed in, the L-register is loaded with 3.4567[1], a copy of X-register contents, before the square is applied. Right after $\boxed{g}\boxed{\sqrt{x}}$, $\boxed{g}\boxed{LSTx}$ retrieves L-register contents back to the X-register[2] so ln(x) can be calculated. What must be remembered is that $\boxed{g}\boxed{LSTx}$ also causes the stack contents to be lifted because $\boxed{g}\boxed{LSTx}$ in fact acts like a number entry, as if 3.4567 is keyed in. The sequence $\boxed{g}\boxed{LN}\boxed{2}\boxed{\times}$(4) computes the second part of the right side of the equation. As it can be seen, the $\boxed{2}$(3) key enters a value in the X-register, so the stack is lifted. Now both x^2 and 3×ln(x) are correctly located in the stack and $\boxed{+}$(5) computes the answer for the y value at once. The diagram below shows each operation and the related stack register contents. a, b, c, d and l are any random values previously in the stack registers. The dashed arrow indicates the first number entry, while straight-line arrows indicate L-register contents change and use.

Figure : Diagram illustrating each operation and related stack register contents



Using the stack registers efficiently does not mean to keep track of all stack register contents all the time. Instead keeping track of what is happening, like which registers have their contents being used, is helpful.

Practice solving problems with stack registers

Example 5

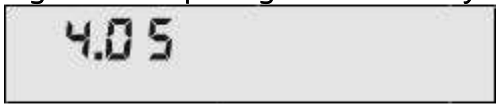
Now what is the shortest keystroke sequence to find 'y' value if the previous expression is changed for the one below, given x=3.4567?

Figure : Equation to find the value of y

$$y = \sqrt{x} + 2 \times \ln(x) - \frac{1}{x}$$

Solution

One of the shortest keystroke sequences to compute 'y' in the expression is:

Keystroke	Display
3 • 4 5 6 7 g √x g LSTx 1/x CHS g LSTx g LN 2 × + +	Figure : Computing the value of y 

In this example, a single shift in the sequence of terms was made. The term 2×ln(x) needs four keystrokes and does not leave a copy of x in L-register, so it was left as the last term to be computed. The expression is written like this:

Figure : Equation to find the value of y

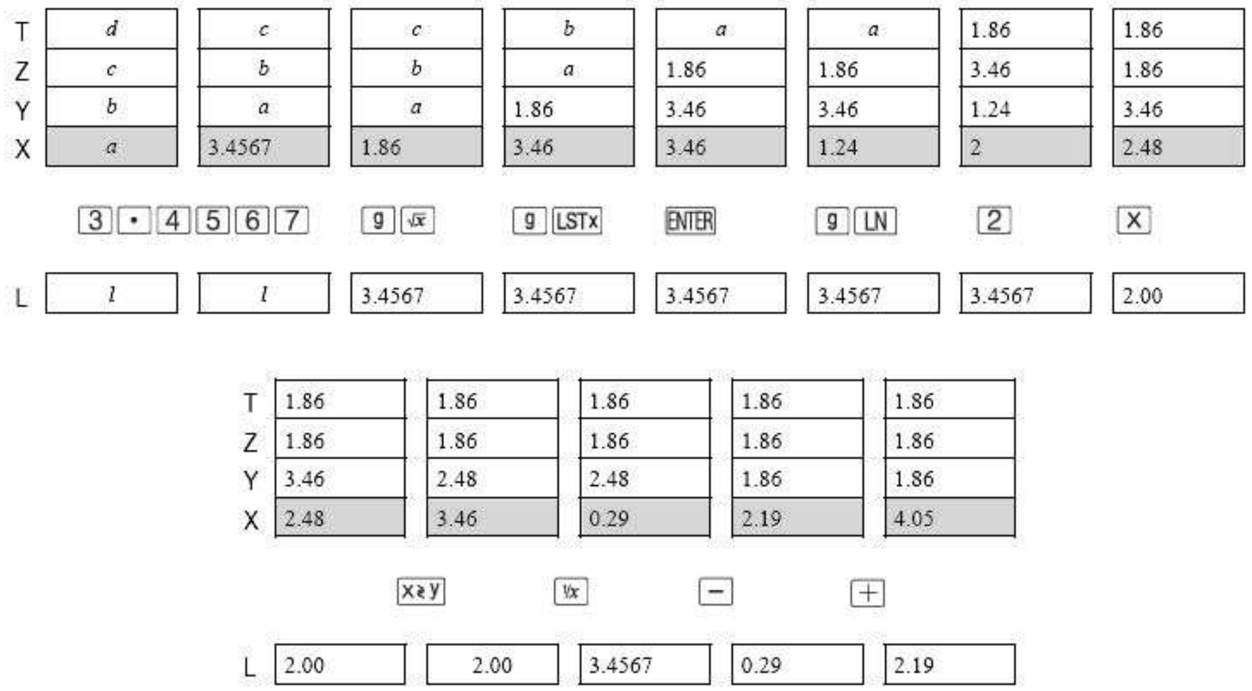
$$y = \sqrt{x} - \frac{1}{x} + 2 \times \ln(x)$$

Another sequence to compute the expression is:

3 • 4 5 6 7 g √x g LSTx ENTER(6) g LN 2 × x≷y(7) 1/x − +

This sequence uses ENTER to duplicate the number in the X-register(6) so it can be used to compute 2×ln(x) while a copy of it held in Y-register is retrieved to compute -1/x later(7).

Figure : Diagram illustrating each operation and related stack register contents



		3 • 4 5 6 7	g √x	g LSTx	ENTER	g LN	2	×
T	*d*	*c*	*c*	*b*	*a*	*a*	1.86	1.86
Z	*c*	*b*	*b*	*a*	1.86	1.86	3.46	1.86
Y	*b*	*a*	*a*	1.86	3.46	3.46	1.24	3.46
X	*a*	3.4567	1.86	3.46	3.46	1.24	2	2.48
L	*l*	*l*	3.4567	3.4567	3.4567	3.4567	3.4567	2.00

		x≷y	1/x	−	+
T	1.86	1.86	1.86	1.86	1.86
Z	1.86	1.86	1.86	1.86	1.86
Y	3.46	2.48	2.48	1.86	1.86
X	2.48	3.46	0.29	2.19	4.05
L	2.00	2.00	3.4567	0.29	2.19

Another possible keystroke sequence to compute y in this example is:

3 • 4 5 6 7 ENTER ENTER g √x x≷y g LN 2 × + x≷y 1/x −

These examples only begin to scratch the surface of the possibilities that exist when using RPN.